Practice Test: Quadratic Formula and Systems of Nonlinear Equations

Name: ______________________

1. Given the function: $f(x) = 2x^2 - 7x$.
   Calculate the rate of change in the function between $x = -3$ and $x = 1$.

Solve the following Quadratic Equations for 'x'.
The answer might be "imaginary", and/or have a square root in it.

2. $6x^2 - 5x - 4 = 0$

3. $x^2 + 2x + 5 = 0$

4. $x^2 - 10x + 25 = 8$

Solve the following Quadratic Equations for 'x'.
The answer might be "imaginary", and/or have a square root in it.

5. $4x^2 + 5x = 3$

6. For the quadratic equations below, state if the answers are "real" or "imaginary", and the exact number of answers.

   a. $9x^2 - 30x + 25 = 0$

   b. $6x^2 - 7x + 10 = 12$

   c. $\frac{1}{2} \cdot x^2 + x + 10 = 4$

7. Consider the quadratic equation: $ax^2 - 12x - 3 = 0$.
   What values or values of 'a' will guarantee that the equation has "two imaginary answers".

8. Consider the quadratic equation: $\frac{3}{4} \cdot x^2 + bx + 8 = 0$.
What values or values of 'b' will guarantee that the equation has "one real answer".

Calculate the point or points of intersection of the following nonlinear equations. If they do not intersect, state why.

9. $y = x^2 + 5x - 30$
$y = 2x - 12$

10. $x^2 - 4x + y = 10$
$y = 3x - 2$

11. $x^2 + 5x + y = 9$
$x - y = 7$

12. $2x^2 - 12x + y = 6$

$3x + 16 = y$

13. $x^2 - x + y = 1$

$2x - y = -6$

14. $x^2 + y^2 = 100$

$x = -6$

Practice Test - Answers:

1. Rate of change = slope = $\frac{Y_2 - Y_1}{X_2 - X_1}$

- $X = -3$; $f(-3) = 2(-3)^2 - 7\cdot(-3)$
  $= 2\cdot 9 - (-21) = 18 + 21 = 39$
- $X = 1$; $f(1) = 2\cdot 1^2 - 7\cdot 1 = 2 - 7 = -5$
- Rate of change $= \frac{39-(-5)}{-3-1} = \frac{44}{-4} = -11$

2. $6X^2 - 5X - 4 = 0$

$a = 6$, $b = -5$, $c = -4$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4\cdot 6\cdot(-4)}}{2\cdot 6} = \frac{5 \pm \sqrt{25 + 144}}{12} = \frac{5 \pm \sqrt{169}}{12} = \frac{5 \pm 13}{12} = :$$

a. $\frac{5+13}{12} = \frac{18}{12} = \frac{3}{2}$ b. $\frac{5-13}{12} = \frac{-8}{12} = \frac{-2}{3}$

3. $X^2 + 2X + 5 = 0$

$a = 1$, $b = 2$, $c = 5$

$$X = \frac{-2 \pm \sqrt{4 - 4\cdot 1\cdot 5}}{2\cdot 1} = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = \frac{-2 \pm 4i}{2} = \frac{-2}{2} \pm \frac{4i}{2} = -1 \pm 2i$$

4. $X^2 - 10X + 25 = 8$

$\quad\quad\quad\quad -8 \quad -8$

$X^2 - 10X + 17 = 0$

$a = 1$, $b = -10$, $c = 17$

$$X = \frac{10 \pm \sqrt{100 - 4\cdot 1\cdot 17}}{2\cdot 1} = \frac{10 \pm \sqrt{100 - 68}}{2} = \frac{10 \pm \sqrt{32}}{2} = \frac{10 \pm \sqrt{16}\sqrt{2}}{2} = \frac{10 \pm 4\sqrt{2}}{2} = 5 \pm 2\sqrt{2}$$

5. $4x^2 + 5x = 3$ — set equal to 0

$$\begin{aligned} 4x^2 + 5x &= 3 \\ -3 \quad & -3 \end{aligned}$$

$4x^2 + 5x - 3 = 0$

$a = 4$
$b = 5$
$c = -3$

$x = \dfrac{-5 \pm \sqrt{25 - 4 \cdot 4 \cdot (-3)}}{2 \cdot 4} = \dfrac{-5 \pm \sqrt{25 + 48}}{8} =$

$\dfrac{-5 \pm \sqrt{73}}{8}$ } This is the answer — can't be simplified.

6. a. $a = 9$; $b = -30$; $c = 25$

Discriminant $= b^2 - 4ac = (-30)^2 - 4 \cdot 9 \cdot (25) = 900 - 900 = 0$

• $b^2 - 4ac = 0$ means: one real answer

b. $6x^2 - 7x + 10 = 12$

$\quad -12 \quad -12$

$6x^2 - 7x - 2 = 0$ — $a = 6$, $b = -7$, $c = -2$ → $b^2 - 4ac =$

$(-7)^2 - 4 \cdot 6 \cdot (-2) =$

$49 + 48 = 97$

• $b^2 - 4ac = 97 > 0$, so there are two real answers.

c. $\frac{1}{2}x^2 + x + 10 = 4$

$\quad -4 \quad -4$

$\frac{1}{2}x^2 + x + 6 = 0$ — $a = \frac{1}{2}$, $b = 1$, $c = 6$ → $b^2 - 4ac =$

$1^2 - 4 \cdot \frac{1}{2} \cdot 6 =$

$1 - 12 = -11$

• $b^2 - 4ac = -11 < 0$, so there are two imaginary answers.

7. "Two imaginary answers" means $b^2 - 4ac < 0$.

$a = a$
$b = -12$
$c = -3$

$(-12)^2 - 4 \cdot a \cdot (-3) = 144 + 12a < 0$

$\quad -144 \qquad -144$

$12a < -144$

$\frac{12a}{12} < \frac{-144}{12}$

$\underline{a < -12}$

8. One real answer means: $b^2 - 4ac = 0$

$$\left.\begin{array}{l} a = 3/4 \\ b = b \\ c = 8 \end{array}\right\} \quad b^2 - 4(3/4)\cdot 8 = b^2 - 24 = 0$$

$$+24 \quad +24$$

$$\sqrt{b^2} = \sqrt{24}$$

$$b = \pm\sqrt{4}\sqrt{6} = \pm 2\sqrt{6}$$

9. $y = x^2 + 5x - 30 = 2x - 12$ ⟶ set equal to 0

$$-2x + 12 \quad -2x + 12$$

$x^2 - 3x - 18 = 0$. Solve using the Quadratic Formula:

$$\left.\begin{array}{l} a = 1 \\ b = -3 \\ c = -18 \end{array}\right\} \quad x = \frac{3 \pm \sqrt{9 - 4\cdot 1\cdot(-18)}}{2\cdot 1} = \frac{3 \pm \sqrt{9+72}}{2} = \frac{3 \pm \sqrt{81}}{2} =$$

$$\frac{3 \pm 9}{2} = \frac{12}{2} = 6, \text{ or } \frac{-6}{2} = -3$$

Got the x-values — need to calculate y-values:

- $x = 6$; $y = 2x - 12 = 2\cdot 6 - 12 = 12 - 12 = 0 \Rightarrow (6, 0)$
- $x = -3$; $y = 2\cdot(-3) - 12 = -6 - 12 = -18 \Rightarrow (-3, -18)$

10. $x^2 - 4x + y = 10$; $y = 3x - 2$

⟶ replace this "y" with "3x-2"

$$x^2 - 4x + 3x - 2 = 10$$

$$x^2 - x - 2 = 10$$

$$-10 \quad -10$$

$$x^2 - x - 12 = 0$$

$$\left.\begin{array}{l} a = 1 \\ b = -1 \\ c = -12 \end{array}\right\} \quad x = \frac{1 \pm \sqrt{1 - 4\cdot 1\cdot(-12)}}{2\cdot 1} = \frac{1 \pm \sqrt{1+48}}{2} = \frac{1 \pm \sqrt{49}}{2} =$$

$$\frac{1 \pm 7}{2} = \frac{8}{2} = 4, \text{ or } \frac{-6}{2} = -3$$

Need to calculate the y-values:

- $x = 4$; $y = 3x - 2 = 3\cdot 4 - 2 = 12 - 2 = 10 \Rightarrow (4, 10)$
- $x = -3$; $y = 3\cdot(-3) - 2 = -9 - 2 = -11 \Rightarrow (-3, -11)$

11. $x^2 + 5x + y = 9$
$\quad\;\; x - y = 7$ } Use "Elimination" — Add

$x^2 + 6x = 16$
$\quad\quad -16 \;\; -16$

$x^2 + 6x - 16 = 0$

$a = 1$, $b = 6$, $c = -16$

$$x = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-16)}}{2 \cdot 1} = \frac{-6 \pm \sqrt{36 + 64}}{2} = \frac{-6 \pm \sqrt{100}}{2} =$$

$$\frac{-6 \pm 10}{2} = \frac{4}{2} = 2, \text{ or: } \frac{-16}{2} = -8$$

Need to calculate the y-values:

$x = 2 \rightarrow x - y = 2 - y = 7$; $y = -5$; Point: $(-2, -5)$

$x = -8 \rightarrow x - y = -8 - y = 7$; $y = -15$; Point: $(-8, -15)$

12. $2x^2 - 12x + y = 6$
$\quad y = 3x + 16 \longrightarrow$ plug in "$3x+16$" in place of 'y':

$2x^2 - 12x + (3x + 16) = 6$

$2x^2 - 9x + 16 = 6$
$\quad\quad\quad -6 \;\; -6$

$2x^2 - 9x + 10 = 0$

$a = 2$, $b = -9$, $c = 10$

$$x = \frac{9 \pm \sqrt{81 - 4 \cdot 2 \cdot 10}}{2 \cdot 2} = \frac{9 \pm \sqrt{81 - 80}}{4} = \frac{9 \pm \sqrt{1}}{4} =$$

$$\frac{9 \pm 1}{4} = \frac{10}{4} = 2.5, \text{ or: } \frac{9 - 1}{4} = 2$$

Need to calculate y-values:

$x = 2.5$; $y = 3x + 16 = 3(2.5) + 16 = 23.5 \longrightarrow (2.5, 23.5)$

$x = 2$; $y = 3 \cdot 2 + 16 = 6 + 16 = 22 \longrightarrow (2, 22)$

13. $x^2 - x + y = 1$ → Solve by adding/using Elimination

$$\begin{array}{r} x^2 - x + y = 1 \\ 2x - y = -6 \\ \hline x^2 + x = -5 \\ +5 \quad +5 \\ \hline x^2 + x + 5 = 0 \end{array}$$

$a = 1$, $b = 1$, $c = 5$

$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-19}}{2} = \frac{-1 \pm i\sqrt{19}}{2}$$

Because there is an "i" in the answer, the problem has "NO SOLUTION" — the line and parabola do NOT intersect.

14. $x^2 + y^2 = 100$ → plug $x = -6$ into the equation:

$$(-6)^2 + y^2 = 100$$

$$36 + y^2 = 100$$

$$-36 \qquad -36$$

$$\sqrt{y^2} = \sqrt{64} \longrightarrow y = 8 \text{ or } -8$$

So, the two points of intersection are: $(-6, 8)$ and $(-6, -8)$.